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Editorial

In this edition of our journal, we spotlight a significant advancement in AC voltage measurement technology. The featured research paper delves into a fast AC voltage RMS measuring converter that incorporates an iterative additive error correction method. This innovative approach promises high conversion rates and precise error analysis, offering substantial improvements in voltage measurement accuracy and speed.

The paper introduces an AC voltage RMS measuring converter that employs an iteratively integrating conversion method. This method enhances the conversion rate, making it highly efficient for various applications. The iterative additive error correction process is at the heart of this converter's design, ensuring that conversion errors are minimized and accuracy is maximized [1].

The featured paper offers a detailed exploration of a fast AC voltage RMS measuring converter with iterative additive error correction. By combining high conversion rates with rigorous error analysis and correction, this research provides valuable insights and advancements in voltage measurement technology. We are excited to share these findings with our readers and anticipate that they will inspire further research and development in this field.

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Fast RMS-to-DC Measuring Converter

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ABSTRACT: Fast AC voltage RMS measuring converter with iterative additive error correction which implements an iteratively integrating conversion method is considered. The use of this converter allows a high conversion rate to be achieved. Error analysis of this converter is described. Basic equations for the determining the convergence condition for the error correction process and the process of establishing the converter output voltage, the absolute and relative conversion errors, as well as the number of conversion cycles at which the conversion error becomes less than the specified one. The possibility of achieving a high conversion speed of the considered converter just due to the use of an iteratively integrating average voltage converter as a low-pass filter is shown. A significant disadvantage of the converter in question is the dependency of the conversion speed on the frequency of the input voltage. This factor limits the possible frequency range. However, if the frequency range is sufficiently narrow or the frequency is constant, a high conversion speed can be achieved in this converter.

KEYWORDS: RMS-to-DC converter, Conversion equation, Absolute and relative conversion errors

1. Introduction

As you know, RMS, average and amplitude voltage measurement methods are commonly used in power electronics for measuring the voltage level of electrical signals [1]. Here's a brief comparison of these methods:

1. Amplitude voltage measurement: This method measures the maximum voltage level of the signal over a period of time. It's a simple method but doesn't take into account the fluctuations of the signal.

2. Average voltage measurement: This method calculates the average voltage level of the signal over a period of time. It's also a simple method, but it doesn't give an accurate representation of the signal's characteristics if the signal has fluctuations.

3. RMS voltage measurement: This method calculates the root-mean-square value of the signal over a period of time. It takes into account the fluctuations of the signal and provides a more accurate representation of the signal's characteristics.

The advantage of using the RMS measurement method is that it gives an accurate representation of the actual power being delivered to a load. This is because the power delivered to a load is proportional to the square of the voltage. The RMS value takes into account the fluctuations of the signal, which means it reflects the true power being delivered to the load. In contrast, the average

or amplitude methods may underestimate or overestimate the actual power being delivered to the load. Therefore, RMS measurement is commonly used in power electronics for accurate power measurement and control.

As you know, increasing the speed of the AC voltage RMS measuring converter can be beneficial in certain applications. The RMS or Root Mean Square voltage is the effective voltage of an AC waveform and is a crucial parameter in power electronics and electrical power systems.

One reason to increase the speed of the AC voltage RMS measuring converter is to improve the accuracy of the measurement. In some applications, such as power system protection, it is important to detect and respond to abnormal voltage conditions quickly to prevent damage to equipment or power outages. A faster RMS converter can provide more timely and accurate information about the voltage waveform, allowing for faster detection and response to abnormal conditions.

Another reason to increase the speed of the RMS converter is to enable real-time monitoring and control of power systems [1].

A fast RMS converter can provide the necessary data for real-time monitoring and control, allowing for more efficient and effective management of power systems.

The significance of RMS as a measurement for the magnitude of an AC voltage is well-established [2]. It can be defined practically or mathematically. In practical terms, the RMS value of an AC voltage corresponds to the amount of heat generated in the same load. For instance, an AC voltage of 1 volt RMS will result in the same level of heat production in a resistor as a 1-volt DC voltage.

The concept of RMS is widely recognized as a crucial parameter for measuring the magnitude of an AC voltage [2]. Mathematically, the RMS value of a voltage is defined by the following formula:

$$U_{RMS} = \sqrt{Avg(U^2)}$$

A technical tool called a measuring RMS-to-DC converter is designed to provide a DC output that is equivalent to the RMS value of an AC or fluctuating DC input. This converter offers a significant advantage over voltage converters that only measure amplitude or average values because it can provide more detailed information about the measured value, particularly in cases where the input voltage waveform is distorted. In the past, early multimeters used simple rectifiers and averaging circuits to measure AC voltage, but they could only read the correct RMS value for a sine wave. In contrast, true RMS-to-DC converters can measure the RMS value of any input voltage waveform, making them a versatile tool for use in various measurement circuits [2, 3].

How RMS-to-DC converters work. In reference [3], the operation of RMS-to-DC converters is explained. These converters utilize an implicit equation to calculate the RMS voltage value. The process of converting the RMS definition into this implicit equation will be described, followed by an explanation of how it is executed in a monolithic RMS-to-DC converter. To begin with, let's consider the definition of the RMS voltage value.

$$U_{RMS} = \sqrt{\frac{1}{T}\int_0^T [U(t)^2]dt} \quad , \quad (1)$$

The given equation relates U_{RMS}, T, and $U(t)$, where U_{RMS} represents the RMS value, T represents the measurement duration, and $U(t)$ represents the

instantaneous voltage which is a function of time but not necessarily periodic. If we square both sides of this equation, we get another equation:

$$U_{RMS}^2 = \int_0^T [U(t)^2]dt \quad . \quad (2)$$

It is possible to estimate the integral by calculating a moving average

$$Avg[U(t)]^2 = \frac{1}{T}\int_0^T [U(t)^2]dt \quad (3)$$

Equation 2 can be simplified as follows

$$U_{RMS}^2 = Avg[U(t)^2] \quad . \quad (4)$$

If we divide both sides by the root mean square voltage (U_{RMS}), we get:

$$U_{RMS} = \frac{Avg[U(t)^2]}{U_{RMS}} \quad . \quad (5)$$

The technique utilized in Analog Devices' monolithic RMS-to-DC converters is based on the implicit solution for U_{RMS}, which is derived from the expression stated. It is important to note that if you take the square root of both sides of Equation 4, the result will follow:

$$U_{RMS} = \sqrt{Avg[U(t)^2]} \quad (6)$$

This is another method of denoting the root mean square value of the function.

An alternative method to express the RMS value of the function is through implicit computation. This method is preferred over the explicit method, which involves successively squaring, averaging, and taking the square root of the input voltage, for practical reasons that yield a better dynamic range. Figure 1 displays the implicit method of converting RMS to DC, which is essentially an analog computer that solves Equation 5.

Variations of this design are utilized in several Analog Devices products including the AD536A, AD636, AD637, AD736, and AD737 [4-7].
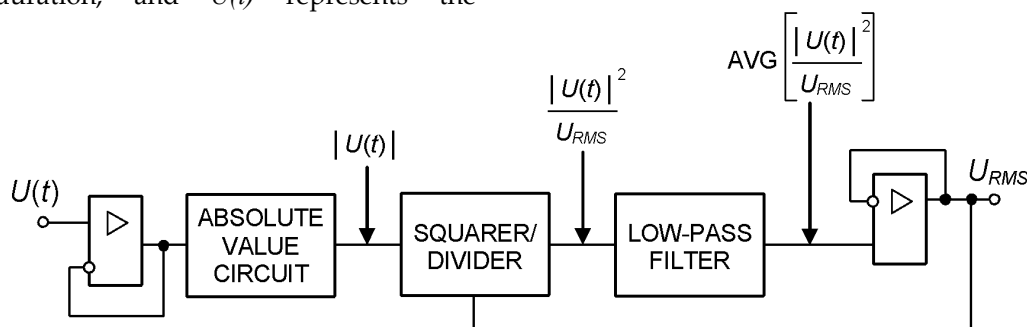

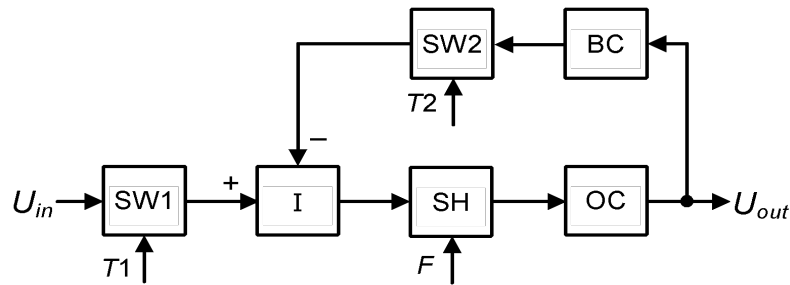Figure 1: Implicit method of RMS-to-DC conversion

Figure 2: Simplified structural scheme of iteratively integrating converter

A precision full-wave rectifier, also known as an absolute-value circuit, is placed after the input buffer, and its output is connected to a squarer/divider. The squarer/divider calculates the input voltage squared and divides it by the averaged output voltage of the squaring circuit. By creating a loop around the divider, Equation 5 can be continuously solved. Speed is a crucial metrological parameter for RMS-to-DC converters, which are extensively utilized in various technological fields.

2. Problem statement

The discussed RMS-to-DC converters have limited performance due to the presence of a low-pass filter, which is necessary to obtain a DC voltage output. This filter often restricts the usability of the converter. To overcome this limitation and increase the speed of the devices, an iterative integrating transformation method can be utilized. This method has been successfully employed in developing high-precision measuring converters, as described in various sources [8–12].

3. Solution of the problem

Iterative methods of error correction based on a well-established mathematical theory hold a significant position among the structural approaches to enhance the precision and speed of measuring converters. Meanwhile, integrating converters are extensively utilized in measurement techniques owing to their numerous benefits such as high accuracy, low susceptibility to interference, sensitivity, simplicity, reliability, cost-effectiveness, and more.

Combining methods of integrating conversion and additive iterative correction of errors has generated a method called by the author in reference [8] iteratively integrating conversion method and devices using this method – iteratively integrating converters.

Measuring iteratively integrating converter. Figure 2 displays a simplified structural scheme of the iteratively integrating converter proposed by the author in reference [8].

Here the input voltage is denoted as U_{in}; the output converter (OC) and inverse converter (BC) which function as scaling converters or amplifiers. The OC and BC transfer coefficients are represented by K_{OC} and K_{BC}, respectively. The system also includes switches SW1 and SW2, sampling and holding circuit (SH). The time interval T1 for the integration of the input voltage Uin and the time interval $T2$ for the integration of the output voltage U_{out}, which are normally regarded as input quantities.

Work occurs in periodic cycles, which are determined by a sequence of sampling pulses denoted as F. Each cycle is a step in the iterative process and is composed of several stages. First, the input voltage Uin is integrated over a period of time $T1$ using an integrator with a non-inverting input. Next, the product of quantities K_{BC} and U_{out} is integrated over a period of time $T2$ using the same integrator, but with an inverting input. The resulting output voltage is then stored by a sample and hold circuit (SH) until the end of the cycle, when it is sampled by the next pulse in the sequence F. Finally, this value is transmitted through an output converter (OC) to the converter output during the subsequent conversion cycle.

Suppose that the output voltage U_{out} is initially equal to U_{out0}. If the input voltage U_{in} undergoes a sudden change, we can express the output voltage for the n-th cycle following this jump in the following manner:

$$U_{out\ n} = U_{in}\ T1\ K_{OC}\ /\ R\ C + U_{out\ (n-1)}\ Q \qquad (7)$$

where $Q = 1 - T2\ K_{OC}\ K_{BC}\ /\ R\ C$; R and C are elements of the integrator I.

The equation (7) is an inhomogeneous linear differential equation of the first order. As is well-known, this type of equation can be solved using methods such as the Z-transform. By applying this method, we obtain a solution.

$$U_{out\ n} = U_{in}\ T1\ K_{OC}\ /\ R\ C\ \sum_{j=1}^{n} Q^{j-1} + U_{out\ 0}\ Q^n. \qquad (8)$$

The variable "j" represents the sequence number of the conversion cycle that follows a change in input value, such as a jump in input voltage (U_{in}), $j = \overline{1, n}$.

An alternative solution to equation (8) is apparent and involves utilizing equation (7) as a recurrence formula to directly generate the series (8). The validity of this approach is established through mathematical induction, although the details of the proof will not be expounded on for the sake of brevity.

The equation (8) denotes the sum of a geometric series with a denominator Q that converges only if $|Q|$ is less than 1. Moreover, the term $U_{out}Q^n$ decreases as the same condition is met.

After applying established formulas for geometric progression and recurrence (formula 8), we can express the output voltage of the converter at the conclusion of the first conversion cycle following an input voltage surge in the following manner:

$$U_{out\,1} = U_{in}\,T1\,K_{OC}\,/\,R\,C + U_{out\,0}\,Q, \qquad (9)$$

at the conclusion of the second conversion iteration

$$U_{out\,2} = (U_{in}\,T1\,K_{OC}\,/\,R\,C)\,(1+Q) + U_{out\,0}\,Q^2, \quad (10)$$

$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots$$

after completing the nth conversion cycle

$$U_{out\,n} = U_{in}\,T1\,K_{OC}\,/\,R\,C \sum_{j=1}^{n} Q^{j-1} + U_{out\,0}\,Q^n =$$
$$= (U_{in}\,T1\,/\,T2\,K_{BC})\,(1 - Q^n) + U_{out\,0}\,Q^n \qquad (11)$$

in steady state ($n\to\infty$)

$$U_{out\,\infty} = \lim_{n\to\infty} U_{out\,n} = U_{in}\,T1\,/\,T2\,K_{BC}. \qquad (12)$$

The conversion equation of the converter in a static state is represented by this expression, while expressions (8) – (11) illustrate the dynamics of the transitional process involved in setting the output value following a sudden alteration in input voltage.

It is possible to express the formula for calculating the relative error γ_n resulting from finite conversion time ($n \neq \infty$) by utilizing equations (8) and (12) in the following manner:

$$\gamma_n = \frac{U_{outn} - U_{out\infty}}{U_{out\infty}} = -\frac{\Delta U Q^n}{U_{out\infty}}, \qquad (13)$$

where $\Delta U = U_{out\infty} - U_{out0}$.

Equation that calculates the cycle count n at which the error decreases below the specified level:

$$n = \left[\frac{\ln\left| \dfrac{\gamma_n U_{out\infty}}{\Delta U} \right|}{\ln|Q|} \right] + 1. \qquad (14)$$

The integer part of the expression within the square brackets is denoted by them. Based on the previous analysis, we can conclude that the closer the Q value is to zero, the faster the transient process will be. When Q equals zero, the transitional process reaches its maximum speed and completes within a single cycle.

The conversion process in the device involves an iteratively additive correction method [8], where the output voltage of the device is used as a test signal. The iterative error correction in the converter gradually brings the output value closer to a predetermined steady-state value. Additionally, the correction is additive, meaning that it is achieved by summing.

The iteratively integrating converter produces an output voltage, U_{out}, that is in direct correlation with the average value of the input voltage, U_{in}, over the duration of the averaging time, $T1$. This characteristic enables the development of a voltage converter that can generate average voltage values using the aforementioned iteratively integrating converter [8,13]. Consider an RMS to DC converter using the iteratively integrated converter described earlier.

Fast RMS-to-DC measuring converter. The high-speed RMS-to-DC measurement converter shown in Figure 3 (simplified block diagram) and Figure 4 (simplified functional diagram) was built using the configuration shown in Figure 1, but with some modifications. In particular, the converter uses only a squarer/divider and an iterative integrator to calculate average voltage values as a low-pass filter. In addition, to simplify the circuit, the circuit for generating the absolute value of the input voltage and the input buffer (i.e. the circuit for converting the input voltage $U_{in}(t)$ to voltage $|U_{in}(t)|$ is not shown.

The RMS-to-DC measuring converter has a distinct design compared to the previously discussed iteratively integrating average voltage converter. This is due to the inclusion of a squarer/divider and a feedback circuit that links the converter output to the control input of the squarer/divider. Prior to being fed into the integrator's input, the input voltage $|U_{in}(t)|$ is subjected to a squaring operation.

The design of the squarer/divider follows the pattern of the widely recognized approximating RMS-to-DC measurement converter. This converter employs operational amplifiers with diodes in the feedback loop to construct the approximation cells.

The simplified block diagram comprises of a squarer/divider which is made up of approximation cells C_1, C_2, ...C_m and an adder Σ. Additionally, there is a low-pass filter which consists of an integrator I, a sample and hold circuit SH, an amplifier A, and a pulse shaper F. It's worth noting that the low-pass filter is the previously discussed iteratively integrating average voltage converter. Each of the approximation cells C1, C2, ...Cm is an inverting scaling converter with an offset that functions in a single quadrant. The first approximation cell C_1 only contains the INV inverter.

The simplified functional diagram features a series of approximation cells. Each of these cells, with the exception of the first cell C_1, functions as an inverting summing scaling converter (amplifier) that is based on an operational amplifier with diodes incorporated into the feedback circuit. The inclusion of diodes within the amplifier allows for a rectifier function to be carried out.
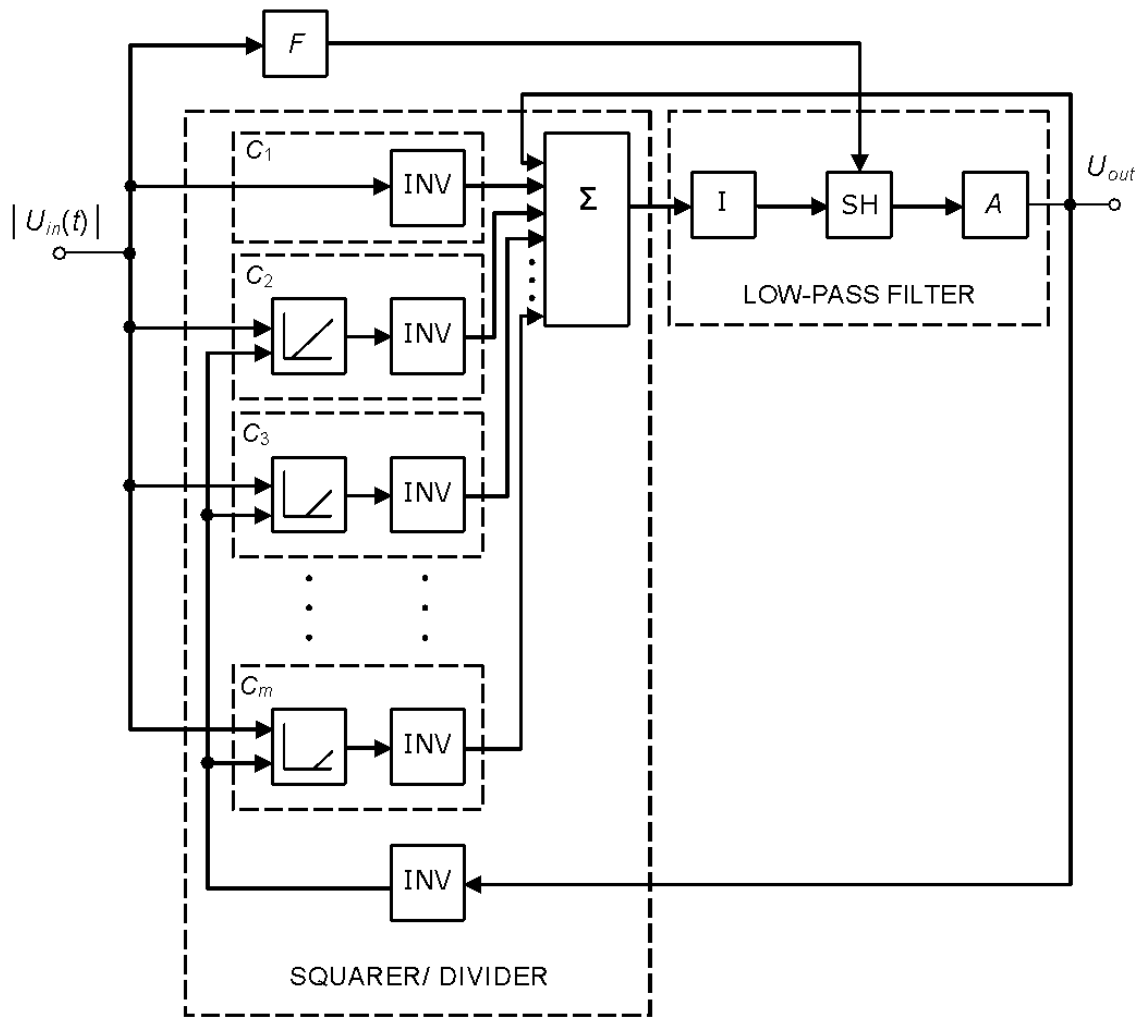
Figure 3: Simplified block diagram of the fast RMS-to-DC measuring converter

As a result, the converter operates as a one quadrant scaling converter. The characteristic of the summing scaling converter, which demonstrates the relationship between output voltage and input voltage, is shifted along the horizontal axis due to two voltages: the input voltage $|U_{in}(t)|$ and the converter output voltage Uout. The extent of this shift is determined by the scale converter resistors and the converter output voltage U_{out}. The response of all scaling cells creates a quadratic relationship, which allows the squarer/divider to perform the function of squaring the voltage at its input. The first approximation cell C1 exclusively features the INV inverter.

The squarer/divider device computes the square of the input voltage's absolute value and then divides it by the converter's output voltage. The latter is determined by averaging the output voltage of the squaring circuit. The transformation function of the squarer/divider has a quadratic dependence that is approximated by a piecewise linear curve.

Typically, an approximation error of no more than 0.1% is achieved by using 5-7 segments of approximation. The slopes of the approximating curve's segments are

determined by the ratio of the resistance values of the corresponding resistors. The input voltage values at which the approximating curve intersects or changes direction are dependent on both the ratio of resistance values of the corresponding resistors and the magnitude of the converted voltage.

The sliding bias, which is the feedback circuit connecting the output of the converter to the control input of the squarer/divider, has two important functions. Firstly, it enables the extraction of the square root from the input voltage $\overline{|U_{in}(t)|}^2$ that has been squared and averaged. Secondly, it helps reduce errors resulting from the piecewise linear approximation of the quadratic curve formed by the cells. This is achieved by adjusting the scale of the curve to match the value of the transformed voltage.

The converter is composed of several components including approximating cells constructed using operational amplifiers and diodes in the feedback circuit, as well as an integrator I, a sample and hold circuit SH, an amplifier A, and an inverter INV.
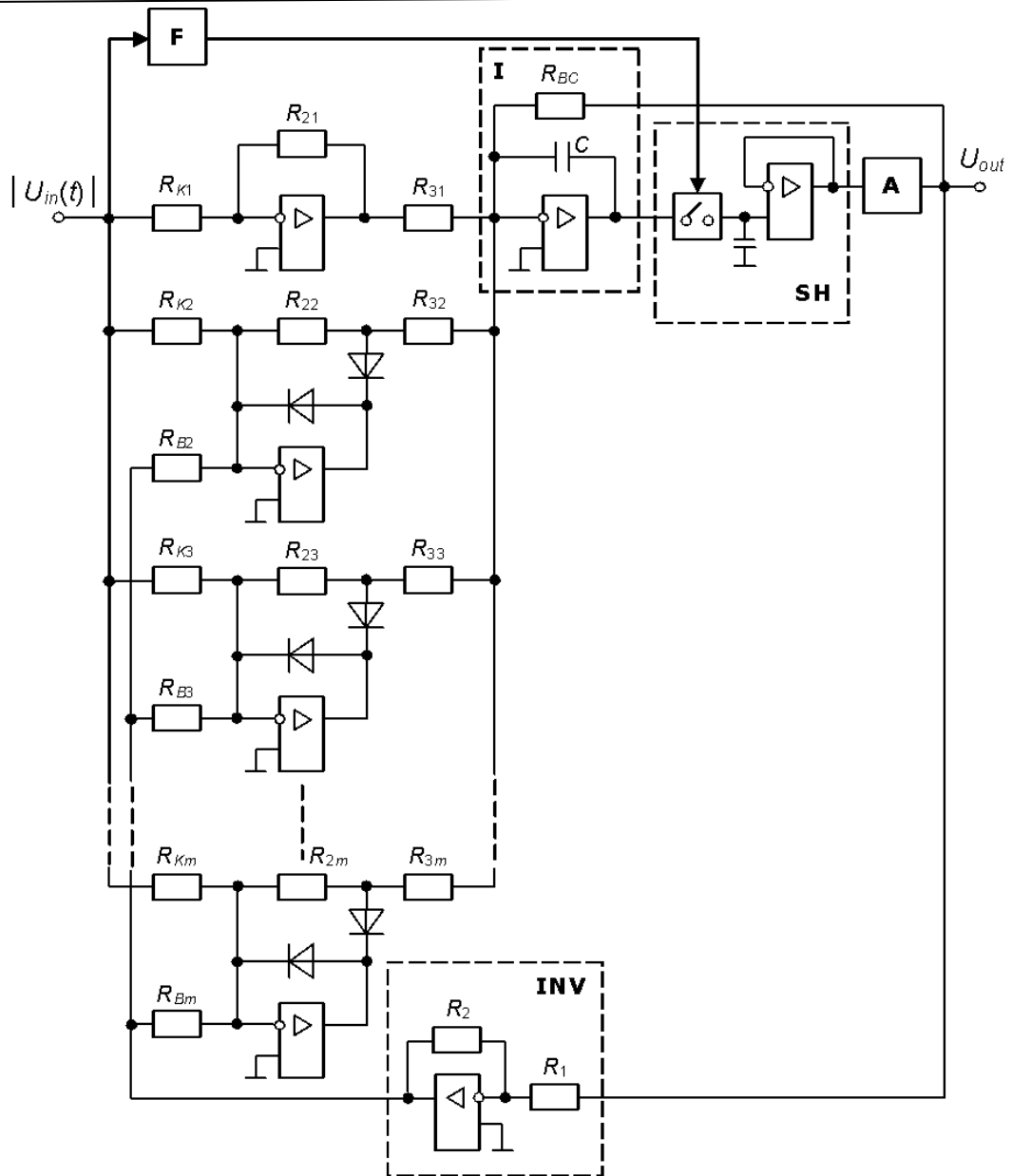
Figure 4: Simplified functional diagram of the fast RMS-to-DC measuring converter

The equations below contain resistances denoted by various indices, such as $R_{K1},...R_{Km}$, $R_{B1},...R_{Bm}$, R_1, R_2, $R_{21},...R_{2m}$, $R_{31},...R_{3m}$, and R_{BC}. These indices correspond to the resistors represented in figure 4. Similarly, the capacitance C in the equations corresponds to the capacitance of the capacitor indicated by the same index in the figure 4 diagram.

The fast converter, which converts the RMS voltage value into a DC voltage value, operates cyclically, similar to the iteratively integrating average voltage converter discussed earlier. The sequence of sample pulses F, which is formed using the pulse shaper F from the input voltage $|U_{in}(t)|$, determines the iteration steps as conversion cycles. The squarer/divider receives input voltage $|U_{in}(t)|$ and squares it. The output voltage of the squarer/divider is then integrated continuously by integrator I, which also

integrates the output voltage of converter U_{out}. Converter output voltage U_{out} is fed to the control input of the squarer/divider as well. At the end of each cycle, when sampling pulse F occurs, the integrator's output voltage is stored by sampling and holding circuit SH. This stored voltage is then passed through amplifier A and fed to the output of converter. As a result, the output voltage of converter U_{out} is equal to the square root of the squared and averaged input voltage $\overline{|U_{in}(t)|^2}$.

Using equations (9) – (14), we can write the following for the RMS-to-DC measuring converter, drawing an analogy with the iteratively integrating converter of average voltage values as discussed earlier.

After the input voltage jump, the voltage produced by the converter at the completion of its first conversion cycle can be represented by the following expression

$$U_{out1} = \{ \frac{TK_{OC}}{C} \{ \overline{U_{in}(t)} \frac{R_{21}}{R_{K1}R_{31}} +$$

$$+ (\overline{U_{in}(t)} \frac{R_{22}}{R_{K2}R_{32}} - U_{out0} \frac{R_2 R_{22}}{R_1 R_{B2} R_{32}})[[Sign(\overline{U_{in}(t)} \frac{R_{22}}{R_{K2}R_{32}} - U_{out0} \frac{R_2 R_{22}}{R_1 R_{B2} R_{32}}) + 1]/2] +$$

$$+ (\overline{U_{in}(t)} \frac{R_{23}}{R_{K3}R_{33}} - U_{out0} \frac{R_2 R_{23}}{R_1 R_{B3} R_{33}})[[Sign(\overline{U_{in}(t)} \frac{R_{23}}{R_{K3}R_{33}} - U_{out0} \frac{R_2 R_{23}}{R_1 R_{B3} R_{33}}) + 1]/2] + \ldots$$

$$\ldots + (\overline{U_{in}(t)} \frac{R_{2m}}{R_{Km}R_{3m}} - U_{out0} \frac{R_2 R_{2m}}{R_1 R_{Bm} R_{3m}})[[Sign(\overline{U_{in}(t)} \frac{R_{2m}}{R_{Km}R_{3m}} - U_{out0} \frac{R_2 R_{2m}}{R_1 R_{Bm} R_{3m}}) + 1]/2] \} + U_{out0} Q_T \} / U_{out0},$$

where the variable "m" represents the number of pieces used for piecewise linear approximation. The transmission coefficients of blocks OC, SH and A are denoted by K_{OC}, K_{SH}, and K_A, respectively; $Q_T = 1 - T K_{OC} /$

Introduce the following notations:

R_{BC} C; T is the period of the input voltage $U_{in}(t)$. To determine the sign of X, the following rule is used: if X is greater than zero, Sign X equals 1; if X is less than or equal to zero, Sign X equals –1.

$$A_1 = \overline{U_{in}(t)} \frac{R_{21}}{R_{K1}R_{31}};$$

$$A_2 = (\overline{U_{in}(t)} \frac{R_{22}}{R_{K2}R_{32}} - U_{out1} \frac{R_2 R_{22}}{R_1 R_{B2} R_{32}}); B_2 = [Sign(\overline{U_{in}(t)} \frac{R_{22}}{R_{K2}R_{32}} - U_{out1} \frac{R_2 R_{22}}{R_1 R_{B2} R_{32}}) + 1]/2;$$

$$A_3 = (\overline{U_{in}(t)} \frac{R_{23}}{R_{K_3}R_{33}} - U_{out1} \frac{R_2 R_{23}}{R_1 R_{B3} R_{33}}); B_3 = [Sign(\overline{U_{in}(t)} \frac{R_{23}}{R_{K3}R_{33}} - U_{out1} \frac{R_2 R_{23}}{R_1 R_{B3} R_{33}}) + 1]/2;$$

$$A_m = (\overline{U_{in}(t)} \frac{R_{2m}}{R_{Km}R_{3m}} - U_{out1} \frac{R_2 R_{2m}}{R_1 R_{Bm} R_{3m}}); B_m = [Sign(\overline{U_{in}(t)} \frac{R_{2m}}{R_{Km}R_{3m}} - U_{out1} \frac{R_2 R_{2m}}{R_1 R_{Bm} R_{3m}}) + 1]/2.$$

Let's now express this in a more concise manner

$$U_{out1} = [\frac{TK_{OC}}{C}(A_1 + A_2 B_2 + A_3 B_3 + \ldots + A_m B_m) + U_{out0} Q_T]/U_{out0},$$

at the conclusion of the second conversion iteration

$$U_{out2} = [\frac{TK_{OC}}{C}(A_1 + A_2 B_2 + A_3 B_3 + \ldots + A_m B_m)(1 + Q_T) + U_0 Q_T^2]/U_{out1},$$

$$\ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots$$

after completing the nth conversion cycle

$$U_{outn} = [\frac{TK_{OC}}{C}(A_1 + A_2 B_2 + A_3 B_3 + \ldots + A_m B_m)\sum_{j=1}^{n} Q_T^{j-1} + U_{out0} Q_T^n]/U_{out(n-1)} =$$

$$= [\frac{1}{R_{BC}}(A_1 + A_2 B_2 + A_3 B_3 + \ldots + A_m B_m)(1 - Q_T^n) + U_{out0} Q_T^n]/U_{out(n-1)}.$$

in steady state ($n \rightarrow \infty$)

$$U_{out\infty} = \lim_{n \to \infty} U_{outn} = \lim_{n \to \infty} [\frac{1}{R_{BC}}(A_1 + A_2 B_2 + A_3 B_3 + \ldots + A_m B_m)(1 - Q_T^n) + U_{out0} Q_T^n]/U_{out(n-1)}.$$

or

$$U_{out\infty} = \sqrt{\frac{1}{R_{BC}}(A_1 + A_2 B_2 + A_3 B_3 + \ldots + A_m B_m)(1 - Q_T^n) + U_{out0} Q_T^n}.$$

An alternate expression for the RMS value of the input voltage $U_{in}(t)$ can be obtained through this equation, which is similar to equation (6) mentioned earlier.

To determine the relative error γ_{nT} caused by the finite transformation time ($n \neq \infty$), neglecting the error of a higher order of smallness, we can derive expression (15) using a similar approach as in expression (13). Similarly, to calculate the number of cycles n required for this error to become less than a given threshold, we can derive expression (16), using a similar approach as in expression (14).

$$\gamma_{nT} = \frac{U_{outn} - U_{out\infty}}{U_{out\infty}} = -\frac{\Delta U Q_T^n}{U_{out\infty}}, \tag{15}$$

$$n = \left\lceil \frac{\ln\left|\dfrac{\gamma_{nT} U_{out\infty}}{\Delta U}\right|}{\ln\left|Q_T^n\right|} \right\rceil + 1. \tag{16}$$

4. Conclusion

1. A significant disadvantage of the converter in question is the dependency of the conversion speed on the frequency of the input voltage. This factor limits the possible frequency range. However, if the frequency range is sufficiently narrow or the frequency is constant, a high conversion speed can be achieved in this converter.

2. When there is a change in input voltage, the process of adjusting the output voltage to the new level is completed in several periods of the input voltage. This speed is greater when the frequency range of the input voltage is narrower. In fact, if the frequency of the input voltage is kept constant, the transient process can be completed in as little as 1-2 periods. So, for example, if the input voltage operates at a frequency of 50 Hz, the transient will conclude within a range of 20 to 40 milliseconds. Using even a first-order low-pass filter will result in a longer setting time compared to this.

3. The analysis reveals that when implementing an iteratively integrating conversion method in a measurement converter with iterative additive error correction, the steady state conversion equation is independent of the conversion coefficients of the direct circuit blocks (I, SH, OC). These blocks only affect the dynamic quality of the converter, thereby enabling the use of low-quality direct circuit blocks while maintaining high precision for the converter as a whole.

4. The rate at which the transient process occurs is influenced by the Q_T value, and it increases when the Q_T value approaches zero. When the Q_T value equals zero, the transient process concludes within a single cycle of conversion.

5. The converter under scrutiny underwent a computer simulation. The converter was tested using different input voltage frequencies and different values of input voltage variation. The simulation findings have thoroughly verified the accuracy of the analysis results.

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Appendix

Table: A tabular representation illustrating the variables employed in the equations.

1. Introduction
Equations 1 – 6: U_{RMS} is the RMS value; T is the duration of the measurement; $U(t)$ is instantaneous voltage, a function of time, but not necessarily periodic.
2. Solution of the problem
Simplified structural scheme of iteratively integrating converter   *Equations 7 – 14:* Q is a denominator of geometric series for simplified structural scheme of iteratively integrating converter; $Q = 1 - T2\, K_{OC} K_{BC} / R\, C$; K_{OC} and K_{BC} are transfer coefficients of blocs OC and BC, respectively; R and C are the resistance and capacitance of the respective integrator I elements;

U_{in} is input voltage;

$U_{out\,0}$ is initial value of output voltage;

$U_{out\,1}$, $U_{out\,2}$,…$U_{out\,n}$ are value of input voltage at the conclusion of the 1-th, 2-th,…n-th conversion iteration (cycle), respectively;

$U_{out\,\infty}$ is value of output voltage in steady state ($n \to \infty$);

j represents the sequence number of the conversion cycle;

n is the number of the conversion cycles;

$U_{out\,n}$ is output voltage for the n-th cycle;

$U_{out\,(n-1)}$ is output voltage for the $(n-1)$-th cycle;

γ_n is relative error resulting from finite conversion time ($n \neq \infty$) for simplified structural scheme of iteratively integrating converter,

$\Delta U = U_{out\infty} - U_{out0}$.

Simplified functional diagram of the fast RMS-to-DC measuring converter

$Q_T = 1 - T\, K_{OC} / R_{BC}\, C$ is a denominator of geometric series for functional diagram of the fast RMS-to-DC measuring converter;

Sign X equals 1 if X is greater than zero;

if X is less than or equal to zero, Sign X equals −1;

m is a number of pieces of piecewise linear approximation;

R_{K1},…R_{Km}, R_{B1},…R_{Bm}, R_1, R_2, R_{21},…R_{2m}, R_{31},…R_{3m}, and R_{BC} are resistances with indices correspond to the resistors represented in figure 4;

C is the capacitance of the respective integrator I element;

γ_{nT} is relative error caused by the finite ransformation time ($n \neq \infty$) for functional diagram of the fast RMS-to-DC measuring converter;

n is the number of the conversion cycles.