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Fuzzy Matrix Theory based Decision Making for Machine Learning

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ABSTRACT: The Fuzzy set theory has numerous real-life applications in almost every field like artificial intelligence, pattern recognition, medical diagnosis etc. There are so many techniques used for solving decision-making problems given by various researchers from time to time. To be able to make consistent and correct choices is the essence of any decision process pervade with uncertainty. Fuzzy matrix theory plays an important role in scientific development under uncertain conditions. Nowadays there are huge varieties of mobile phones with varying features available in the market. Everyone wants to purchase such a mobile phone which has as many features as possible within it but under his/her budget. This has become an important issue in this modern era where everyone wants to have the most preferred mobile handsets for himself/herself as compared to others. So, in this paper, the Fuzzy matrix approach is used in a decision-making problem where a number of buyers can be able to choose their preferred mobile phones with varying features.

KEYWORDS: Decision Making, Fuzzy Logic Technique, Fuzzy Sets, Max-Min Composition, Fuzzy Matrix Multiplication

1. Introduction

Fuzzy set theory tries to follow more closely the vagueness that is inherent in most natural language and in decision-making processes. In a conventional logic approach, this inherent fuzziness of membership and categorization is not included. Fuzzy logic has found many real-world applications that involve imitating or modeling human behavior for decision making in the real world. The dynamics and complexity of social systems are being explained and modeled through the use of fuzzy set theory. In recent years, computational intelligence has been used to solve many complex problems by developing intelligent systems. Fuzzy logic has proved to be a powerful tool for decision making systems, such as expert systems and pattern classification systems [1, 2]. In [3], the author first introduced the concept of fuzzy matrices, and discussed the convergence of powers of fuzzy matrix. Matrices whose matrix operation defined by fuzzy logical operations and whose entries lies in [0, 1] are called fuzzy matrices. Every fuzzy matrix is a matrix but not conversely. Fuzzy matrices play a fundamental role in fuzzy set theory. They provide us a rich framework within which many problems of practical applications of the theory can be formulated. Fuzzy matrices can be successfully used when fuzzy uncertainty occurs in a problem. These results are extensively used for cluster analysis and classification problems of static patterns under subjective measure of similarity [4]. The theories of fuzzy matrices were developed in [5] as an extension of Boolean matrices. With max-min operation the fuzzy algebra and its matrix theory are considered by many authors [6-11]. In [12] the author studied the canonical form of a transitive fuzzy matrix. In [13], the author presented the canonical form of a strongly transitive matrix. In [14], the author studied the controllable fuzzy matrix. In [15], the author presented some properties of the min-max composition of fuzzy matrices. In [16], the author presented some important results on determinant of square fuzzy matrices. In [17], the author investigated iterates of fuzzy circulant matrices. Determinant theory, powers and nilpotent conditions of matrices over a distributive lattice are considered in [18, 19]. As for as the decision making is concern, it is very important in terms of machine learning or imparting artificial intelligence into machines which work upon the traditional logic theory. It is a process which helps a machine to think like a human being and for a human being to ease his/her difficult decision making process. In this paper fuzzy matrices and fuzzy logic techniques are utilized in order to ease the process of decision making.

2. Review of literature

Lotfi A. Zadeh in 1965 first introduced the concept fuzzy sets in one of his research papers under the heading fuzzy logic or Fuzzy Sets, where he proposed that some ideas either belongs to or does not belongs to in the area of general consideration which disproves the traditional assumptions [20, 21]. Various researchers have used fuzzy set theory to prove real life decision making problems having uncertain information. Fuzzy matrices introduced by M. G. Thomason have become popular in the last two decades. Fuzzy matrix which is an extension of classical matrix with entries lies between 0 and 1. Numerous scholars have proposed new methods in fuzzy matrix and these methods lead fuzzy matrix a new area of fuzzy mathematics in [22] the authors gave a brief concept of Basic matrix theory and fuzzy matrix theory in their book "Elementary Fuzzy matrix theory and fuzzy models for social scientists". Madhumangal Pal introduced two new binary fuzzy operators, ⊕ and ⊙, for fuzzy matrices, he presented several properties on these new binary operations. He also proposed some results on existing operators along with these new operators. In [23] the author discussed Fuzzy Matrix Theory and applied it for recognizing the Qualities of Effective Teacher. They have used FRM (Fuzzy relational maps) in their research to determine the components of teacher quality and to apply the Teacher Quality Index (TQI) for recognizing the qualities of effective teacher, as well as for the developments of educational institutes. In [24] the author describes Fuzzy Symmetric Solutions of Fuzzy Matrix equations by using system of linear equations according to the Kronecker product of matrices. Fuzzy relations are fuzzy subsets of the Cartesian product of two universes of discourse say X and Y these relations are combined with each other by the operation called Composition operation. Various composition operations (methods) available are max-min operation, max-product operation, Min-max composition operation, max-max composition operation, min-min composition operation, max-average composition operation and sum-product composition operation. But max-min composition method is best known in fuzzy logic applications.

3. Preliminaries

3.1. Fuzzy matrix

A matrix $A = [a_{ij}]_{mxn}$ is said to be a fuzzy matrix of order mn, where $a_{ij} \in [0, 1]$, $1 \le i \le m$ and $1 \le j \le n$. For example

$$A = \begin{pmatrix} 1 & 0.4 & 0.2 & 0.1 & 0.9 \\ 0.7 & 0 & 0.7 & 0.9 & 0 \\ 0.3 & 0.4 & 0.8 & 0.1 & 0.5 \\ 0.5 & 0.3 & 0.2 & 0.3 & 0.4 \end{pmatrix}$$ is a 4 × 5 fuzzy matrix.

3.2. Fuzzy composition operation

Suppose $\tilde{R}$ be a fuzzy relation on the Cartesian product space X×Y, $\tilde{S}$ be a fuzzy relation on space Y× Z and $\tilde{T}$ be a fuzzy relation on space X× Z, then fuzzy max-min and max- product is defined in terms of set and membership function notation as [1]:

$$\tilde{T} = \tilde{R} * \tilde{S}$$

$$\mu_{\tilde{T}}(x, z) = \underbrace{\max}_{y \in Y} \{\min (\mu_{\tilde{R}}(x, y), \mu_{\tilde{S}}(y, z))\}$$

$$= \max \{(\mu_{\tilde{R}}(x, y)\}, \text{ if } \mu_{\tilde{R}}(x, y) < \mu_{\tilde{S}}(y, z)$$

$$= \mu_{\tilde{R}}(x, y).$$

This operation is one of the common and best among all the composition operations.

3.3. Multiplication of Fuzzy Matrices s

Let $A = [a_{ij}]_{mxn}$, where $a_{ij} \in [0, 1]$, $1 \le i \le m$ and $1 \le j \le n$, and $B = [b_{jk}]_{nxp}$, where $b_{jk} \in [0, 1]$, $1 \le j \le n$ and $1 \le k \le p$, be two fuzzy matrices then $AB = [C_{ik}]_{mxp}$ is also a fuzzy matrix with respect to the operations defined by max-min operation or min-max operation.

For example, $A = \begin{pmatrix} 0 & 0.9 & 0.7 \\ 0.5 & 0.6 & 0.1 \\ 0.8 & 0.4 & 0.9 \\ 0.3 & 0.7 & 0.2 \\ 0.4 & 1 & 0.3 \end{pmatrix}$ be a 5× 3 fuzzy matrix.

$$B = \begin{pmatrix} 1 & 0.2 & 0.4 & 0.5 \\ 0.4 & 0.3 & 0.8 & 0 \\ 0 & 0.2 & 0.6 & 0.7 \end{pmatrix}$$ be a 3× 4 fuzzy matrix.

Then AB is defined using max-min operation as

$C = AB = [C_{ik}]_{5x4}$

where C_{11} = max {min (0, 1), min (0.9, 0.4), min (0.7, 0)} = max {0, 0.4, 0} = 0.9

C_{12} = max {min (0, 0.2), min (0.9, 0.3), min (0.7, 0.2)} = max {0, 0.3, 0.2} = 0.3

C_{13} = max {min (0, 0.4), min (0.9, 0.8), min (0.7, 0.6)} = max {0, 0.8, 0.6)} = 0.8

And so on.

$$\text{Thus } C = \begin{pmatrix} 0.9 & 0.3 & 0.8 & 0.7 \\ 0.5 & 0.1 & 0.6 & 0.5 \\ 0.8 & 0.2 & 0.4 & 0.7 \\ 0.4 & 0.3 & 0.7 & 0.3 \\ 0.4 & 0.3 & 0.8 & 0.4 \end{pmatrix}$$

4. Decision making by using multiplication of fuzzy matrix

Consider a case in which a person needs to decide related to purchasing a mobile phone with specific highlights that he fundamentally needs among a list of numerous such things with discernable yet similarly contending features and henceforth he/she faces a difficulty in the decision-making that which one should he/she really purchase. In such case, a fuzzy matrix multiplication approach based on max-min operation is utilized for such a decision making problem. In spite of a commitment in machine learning, it can be used to rank the multi-dimensional options of an option set. It is frequently used as a part of engineering for settling on design choices, however, can likewise be used to rank speculation alternatives, seller choices, item choices or some other set of multidimensional substances. The proposed decision-making process in view of a fuzzy matrix approach is described as follows.

Let us suppose that B = $\{b_1, b_2 \ldots b_n\}$ be the set of buyers and M = $\{m_1, m_2 \ldots m_k\}$ be the set of alternatives and each alternative is associated with a set of features defined as F = $\{f_1, f_2 \ldots f_p\}$. The main aim is to provide buyer the best option according to his/her choice. Furthermore the two matrices, BF (buyer features) and FM (mobile features) are constructed. The max-min composition operation is utilized in matrix multiplication method in order to find the BM (buyer mobile) matrix in which the highest value corresponding to the row gives the most appropriate alternative (choice) to the buyer.

4.1. Construction of BF and FM matrices for Decision Making

4.1.1. The BF Matrix:

In order to construct the BF matrix the survey is conducted by asking different questions to the buyers about their choices in terms of features, and converted their choices into membership values, also consider some mobile phones with varying features. Furthermore organize the data collected from the survey into membership valued matrices. Also inquiries have been made to the purchaser about his decision regarding features of the mobiles and converted their choice into membership values using fuzzy logic. Thus, BF matrix is arranged containing buyers along rows and feature choices along columns as shown below.

$$BF = \begin{pmatrix} & f1 & f2 & \ldots & fp \\ b1 & . & . & . & . \\ b2 & . & . & . & . \\ . & . & . & . & . \\ bn & . & . & . & . \end{pmatrix}$$

4.1.2. The FM Matrix

Secondly, reviews about the most common reference mobile phones with varying features are collected, such that each reference mobile phone in terms of its features in membership valued sets are ranked, thus arranging the FM matrix in such a way that the reviewed features are taken along rows and reference mobile phones are taken along columns as shown below.

$$FM = \begin{pmatrix} & m1 & m2 & \ldots & mk \\ f1 & . & . & . & . \\ f2 & . & . & . & . \\ . & . & . & . & . \\ fp & . & . & . & . \end{pmatrix}$$

4.1.3. The BM Matrix

Finally, matrix multiplication based on max-min composition operation is performed between matrix BF and matrix FM to get the resultant matrix BM as shown below.

$$BM = \begin{pmatrix} & m1 & m2 & \ldots & mk \\ b1 & . & . & . & . \\ b2 & . & . & . & . \\ . & . & . & . & . \\ bn & . & . & . & . \end{pmatrix}$$

We will follow the technique that the maximum value along the row corresponding to the buyer in the multiplication matrix gives most appropriate result (mobile phone). Note that the similar questionnaire was accommodated for the overview survey as well as for the purchaser.

Table 1: Mobile Phones

S.No	M	Mobiles Phones
1	m_1	Samsung
2	m_2	Apple
3	m_3	Google Pixel
4	m_4	Xiaomi
5	m_5	Oppo
6	m_6	One Plus
7	m_7	Motorola
8	m_8	Sony
9	m_9	Lenovo
10	m_{10}	Realme

Table 2: Features

S.No	M	Mobiles Phones
1	f_1	Battery Life
2	f_2	Memory (RAM-ROM)
3	f_3	Design and Build quality

4	f_4	Screen
5	f_5	Camera
6	f_6	Processor Power
7	f_7	Price
8	f_8	Operating System

5. Analysis and Results

A The survey is conducted on 8 measure features of mobile phones such as Battery life, Memory (Ram-Rom), Design and builds quality, Screen, Camera, Processor Power, Price and Operating system. The examples of mobile phones considered for this survey are Samsung, Apple, Google, Realme, Oppo, One plus, Motorola, Sony, Lenovo, and Huawei. Depending upon how much preference is to be given to each asset, in a percentage score from 0 to 100(basically in a range). Data will then be modified from a scale of 0-100 to a scale of 0-1, to avoid any inconvenience regarding multiplication of two matrices, and such that the whole operation results into a binary operation. To the test the model we have considered 10 buyers represented as B = {b_1, b_2, b_3, b_4, b_5, b_6, b_7, b_8, b_9, b_{10}}, 10 mobile phones represented as M = {m_1, m_2, m_3, m_4, m_5, m_6, m_7, m_8, m_9, m_{10}} and 8 features of mobile phones represented as F= {f_1, f_2, f_3, f_4, f_5, f_6, f_7, f_8}. Table 1 and 2 shows mobile phones and their features considered for the survey respectively.

5.1 Questionnaire used to conduct the survey

A questionnaire is designed which is used to conduct the survey among buyers before decision making and the same is used to conduct a survey among the reviewers which can be shown in table 3.

5.2 Analyzing the survey

In the survey we had asked questions to the buyers about their choices in terms of features, and converted the buyer's choice into values between 0-1, and then we have taken some mobile phones as references, now what we have done is that we organized the data collected through survey into 0-1 valued matrices. First, BF matrix containing buyers in rows and feature choices in columns, further we have collected reviews about the most common reference mobile phones, such that we can rank each reference mobile phone in terms of its features in 0-1 valued sets, thus arranging the 2nd matrix FM as reviewed features in rows and reference mobile phones in columns, and finally, we have performed fuzzy matrix multiplication method to get the matrix BM.

A relation matrix between buyers and features BF is created as:

$$BF = \begin{pmatrix} 0.7 & 0.2 & 0.6 & 0.8 & 0 & 0.1 & 0.4 & 0.5 \\ 0.2 & 0.8 & 0.3 & 0.4 & 0.1 & 0.6 & 0.3 & 0.9 \\ 0.5 & 0.6 & 0.4 & 0.6 & 0.9 & 0.4 & 0.5 & 0.8 \\ 0.6 & 0.9 & 0.9 & 0.5 & 0.3 & 0.2 & 0.4 & 0.6 \\ 0.3 & 0.8 & 0.7 & 0.2 & 0.8 & 0.9 & 0.6 & 0.2 \\ 0.6 & 0.1 & 0.8 & 0.4 & 0.2 & 0.1 & 0.8 & 0.4 \\ 0.1 & 0.2 & 0.6 & 0.3 & 0.5 & 0.7 & 0.9 & 0.9 \\ 0.7 & 0.3 & 0 & 0.5 & 0.6 & 0.9 & 0.4 & 0.2 \\ 0.9 & 0.4 & 0.7 & 0.1 & 0.5 & 0.4 & 0.3 & 1 \\ 0.6 & 0.5 & 0.4 & 0.8 & 0.6 & 0.3 & 0.1 & 0.9 \end{pmatrix}$$

Further a relation matrix between features and mobile phones is created as:

$$FM= \begin{pmatrix} 0.4 & 0 & 0.4 & 0.1 & 0.1 & 0.9 & 1 & 0.9 & 0.1 & 0.1 \\ 0.3 & 0.2 & 0.3 & 0.3 & 0.3 & 0.7 & 1 & 0.3 & 0.2 & 0.2 \\ 1 & 0.9 & 0.6 & 0.2 & 0.2 & 0.4 & 0.3 & 0.4 & 0.4 & 0.9 \\ 0.3 & 0.8 & 0.7 & 0.1 & 0.2 & 0.5 & 0.2 & 0.1 & 0.5 & 0.7 \\ 0.9 & 0.3 & 0.5 & 0.9 & 0.4 & 0.7 & 0.7 & 0.2 & 0.9 & 0.5 \\ 0.5 & 0.7 & 0.9 & 0.8 & 0.1 & 0.9 & 0.5 & 0.4 & 0.3 & 0.2 \\ 0.4 & 0.6 & 0.3 & 0.5 & 0.9 & 0.2 & 0.1 & 0.5 & 0.2 & 0.3 \\ 0.3 & 0.4 & 0.7 & 0.6 & 0.5 & 0.4 & 0.9 & 0 & 1 & 0.4 \end{pmatrix}$$

The resultant multiplication matrix BM is obtained using max-min operation as:

$$BM = \begin{pmatrix} 0.6 & 0.8 & 0.7 & 0.5 & 0.5 & 0.7 & 0.7 & 0.7 & 0.5 & 0.7 \\ 0.5 & 0.6 & 0.7 & 0.6 & 0.5 & 0.7 & 0.9 & 0.4 & 0.9 & 0.4 \\ 0.4 & 0.6 & 0.7 & 0.6 & 0.5 & 0.7 & 0.7 & 0.5 & 0.9 & 0.6 \\ 0.9 & 0.5 & 0.6 & 0.6 & 0.5 & 0.7 & 0.9 & 0.6 & 0.9 & 0.6 \\ 0.8 & 0.7 & 0.9 & 0.8 & 0.6 & 0.9 & 0.8 & 0.5 & 0.8 & 0.7 \\ 0.8 & 0.8 & 0.6 & 0.5 & 0.8 & 0.6 & 0.6 & 0.6 & 0.4 & 0.8 \\ 0.6 & 0.7 & 0.7 & 0.7 & 0.9 & 0.7 & 0.9 & 0.5 & 0.9 & 0.6 \\ 0{,}6 & 0.7 & 0.9 & 0.8 & 0.4 & 0.9 & 0.7 & 0.7 & 0.6 & 0.5 \\ 0.7 & 0.7 & 0.7 & 0.6 & 0.5 & 0.9 & 0.9 & 0.9 & 1 & 0.7 \\ 0.6 & 0.8 & 0.7 & 0.6 & 0.5 & 0.6 & 0.9 & 0.6 & 0.9 & 0.7 \end{pmatrix}$$

From above relation matrix BM we conclude that the best choice of buyer b_1 is mobile m_2. Also best choice of buyer b_2 is m_7 and m_9 mobile phones. This means that buyer b_1 prefers apple than the other mobile phones and buyer b_2 prefers Motorola and Lenovo mobiles phones.

5.3 Survey

Below mentioned in table 4 is the survey which is obtained by asking questions to different persons in the Questionnaire.

6. Conclusion

In this paper we have used fuzzy matrix multiplication method for the purpose of decision making. We have considered an example of decisions making process to buy a mobile phone among a list of mobile phones and which are evaluated by eight features. Also we have conducted a survey among the buyers and reviewers using a same questionnaire. The application of fuzzy matrix multiplication approach is utilized here to construct a BF and FM matrices which is further utilized to analyze the preference of buyers among alternative mobile phones.

Table 3: Questionnaire

Questionnaire

Name

Address

Tick the mobile phones do you own:

1. Samsung……. 2. Apple……. 3. Google pixel………. 4. Xiaomi…….. 5. Oppo……….

6. One plus……. 7. Motorola…… 8. Sony……… 9. Lenovo……… 10. Realme……. 11. Other……..

1. Price of mobile phone (thousands):

(a) 1 – 10 …… (b) 10 – 20 ……. (c) 20 – 30 …..…… (d) 30 – 40 ……… (e) 40 – 50……. (f) 50 - 60……. (g) 60 - 70……..

2. Battery life (months):

(a) 6 – 12 …….. (b) 12 – 18 …..….. (c) 18 – 24 …..….. (d) 24 – 30 …..….. (e) 30- 36 …….. (f) More …..….

3. Memory:

(a) 0 – 20 ……….. (b) 20 – 40 …..….. (c) 40 – 60 …..….. (d) 60 – 80 …….. (e) 80 - 100 ……….

4. Design and build quality:

(a) 0 – 2 …….. (b) 2 – 4 …..….. (c) 4 – 6…….. (d) 6 – 8…….. (e) 8 - 10 …………….

5. Screen:

(a) 0 – 20….. (b) 20 – 40….. (c) 40 – 60…… (d) 60 – 80……

6. Camera:

(a) 0 – 20….. (b) 20 – 40…… (c) 40 – 60…… (d) 60 – 80…… (e) 80 - 100 ………

7. Processor Power:

(a) 0 – 5…… (b) 5 – 10…….. (c) 10 – 15…….. (d) 15 – 20…… (e) 20 – 25……..

8. Operating System:

(a) 0 – 20 ………. (b) 20 – 40 …..…… (c) 40 – 60 ………… (d) 60 – 80 …..……. (e) 80 – 100 …….……

Sig. Date.

Table 4: Survey

NAME	MOBILE PHONE	MOBILE PHONE FEATURES							
		PRICE	BATTERY LIFE	MEMORY	BUILD AND DESIGN QUALITY	SCREEN	CAMERA	PROCESSER POWER	OPERATING SYSTEM
ZAINAB JAVAID	APPLE	0.8	0.7	0.8	1	0.9	0.8	0.8	0.9
MOHAMMMAD UZAIR	GOOGLE	0.8	0.9	0.8	0.9	1	0.8	0.9	0.8
ZAID BIN NISAR	SAMSUNG	0.9	0.8	1	0.5	0.6	0.7	0.8	0.8
HANNAN SAJAD	OPPO	0.4	0.3	1	0.3	0.9	0.5	0.4	0.3
TARIQ AHMAD	XIAOMI	0	0.2	0.9	0.8	0.3	0.7	0.6	0.4
SAJAD AHMAD	LENOVO	0.4	0.3	0.6	0.7	0.5	0.9	0.3	0.7
MAJID ISHAQ	ONE PLUS	1	1	0.3	0.2	0.7	0.5	0.1	0.9
RAKESH KUMAR	SAMSUNG	0.1	0.3	0.2	0.1	0.9	0.8	0.5	0.6
MONU SINGH	MOTOROA	1	0.5	0.6	0.3	0.2	0.7	0.9	0.6
INAM ILLAHI	XIAOMI	0.1	0.3	0.2	0.2	0.4	0.1	0.9	0.5
ZUBAIR AHMAD	GOOGLE	1	0.7	0.5	0.9	0.2	0.1	0	1
NEELA SHUKLA	SAMSUNG	0.4	0.3	0.6	0.7	0.2	0	1	0.4
MUDASIR AHMAD	XIAOMI	0.9	0.7	0.4	0.5	0.7	0.9	0.2	0.4
FAYAZ AHMAD	OPPO	0.4	0.3	1	0.3	0.9	0.5	0.4	0.3
BASIT RIYAZ	SAMSUNG	0	0.2	0.9	0.8	0.3	0.7	0.6	0.4
TANVEER JAN	GOOGLE	0.4	0.3	0.6	0.7	0.5	0.9	0.3	0.7
BHAWNA	APPLE	1	1	0.3	0.2	0.7	0.5	0.1	0.9
HUDAYA	REALME	0.1	0.3	0.2	0.1	0.9	0.8	0.5	0.6
RAVI KUMAR	OTHER	1	0.5	0.6	0.3	0.2	0.7	0.9	0.6
NEELIMA	APPLE	0.1	0.3	0.2	0.2	0.4	0.1	0.9	0.5
ADIBA QURESHI	OPPO	1	0.7	0.5	0.9	0.2	0.1	0	1
BASARAT	OTHER	0.4	0.3	0.6	0.7	0.2	0	1	0.4
DANISH	XIAOMI	0.9	0.7	0.4	0.5	0.7	0.9	0.2	0.4
NASREEN NOOR	SONY	0.6	0.1	0.8	0.4	0.2	0.1	0.8	0.4
JYOTSNA SAHU	XIAOMI	0	0.9	0.8	0.6	0.4	0.2	0.8	0.1
MEHRUNISA KHAN	SONY	0.1	0.2	0.3	0.4	0.6	0.9	0.5	0.7
SHAHEEN SHIKH	APPLE	0.4	0.3	0.6	0.7	0.2	0	1	0.4
SARWAT ALAM	SAMSUNG	0.9	0.7	0.4	0.5	0.7	0.9	0.2	0.4
REHMAT ANSARI	OPPO	0.6	0.1	0.8	0.4	0.2	0.1	0.8	0.4
PRIYA DAVE	REALME	0.9	0.8	0.6	0.5	0.8	0.8	0.4	0.1

DEEPAK KUMAR	GOOGLE	0.4	0.5	0.9	0.5	0.6	0.8	0	1
KHALID AHMAD	SAMSUNG	0.6	0.5	0.4	0.8	0.6	0.3	0.1	0.9
BHAWNA KASBE	GOOGLE	0.1	0.3	0.2	0.1	0.9	0.8	0.5	0.6
TABASUM	APPLE	1	0.5	0.6	0.3	0.2	0.7	0.9	0.6
SUMIRA	REALME	0.5	0.6	0.4	0.6	0.9	0.4	0.5	0.8
ZEBA PATEL	OTHER	02	0.3	0.8	0.5	0.7	0.61	0	0.9
ASHIQ HUSSAIN	APPLE	0.2	0.8	0.3	0.4	0.1	0.6	0.3	0.9
ARSHID AH.	OPPO	0	0.8	0.7	0.9	1	0.5	0.7	0.5
PRITI SHUKLA	OTHER	0.1	0.3	0.2	0.1	0.9	0.8	0.5	0.6
MUZAFAR AH.	XIAOMI	1	0.5	0.6	0.3	0.2	0.7	0.9	0.6
RIKAZ RATHORE	SONY	0.9	0.3	0.4	0.1	0.2	0.4	0.5	0
GITABAI	XIAOMI	1	0.9	0.7	0.5	0.4	0.3	0.2	0.4
GEETA RAO	SONY	0.1	0.3	0.2	0.1	0.9	0.8	0.5	0.6
SARIKA SHARMA	APPLE	1	0.5	0.6	0.3	0.2	0.7	0.9	0.6
TARIQ AH.	SAMSUNG	0.1	0.2	0.4	0.5	0.9	0.3	0.2	1
NIHARIKA TIWARI	OPPO	0.1	0.3	0.2	0.1	0.9	0.8	0.5	0.6
SUSHMA SINGH	REALME	1	0.5	0.6	0.3	0.2	0.7	0.9	0.6
CHITRA SONI	GOOGLE	1	0.5	0.6	0.3	0.2	0.7	0.9	0.6
POOJA TIWARI	XIAOMI	0.1	02.	0.6	0.3	0.5	0.7	0.9	0.9
SUSHILA ATHYA	SONY	0.3	0.8	0.7	0.2	0.8	0.9	0.6	0.2
SEEMA KHAN	OPPO	0.8	0.5	0.3	0.2	0.8	0.8	0.7	0.3
ILYAS AH.	MOTOROLA	1	0.5	0.6	0.3	0.2	0.7	0.9	0.6
SHOWKAT	SAMSUNG	0.1	0.2	0.9	0.7	0.5	0.2	0.3	0.4
S. PAWAR	XIAOMI	0.1	0.3	0.2	0.1	0.9	0.8	0.5	0.6
NIDHI KHATRI	OPPO	1	0.5	0.6	0.3	0.2	0.7	0.9	0.6
MEHZABI ANSARI	SAMSUNG	0.9	0.4	0.7	0.1	0.5	0.4	0.3	1
MAJID ANSARI	GOOGLE	0.3	0.8	0.7	0.2	0.8	0.9	0.6	0.2
NAMRATA JI	APPLE	0.8	0.5	0.3	0.2	0.8	0.8	0.7	0.3
DOLLY	REALME	0.3	0.8	0.7	0.2	0.8	0.9	0.6	0.2
PRINCE	OTHER	0.8	0.5	0.3	0.2	0.8	0.8	0.7	0.3
RASHMI	APPLE	1	0.5	0.6	0.3	0.2	0.7	0.9	0.6
SHWETA JAIN	OPPO	0.6	0.9	0.9	0.5	0.	0.2	0.4	0.6
DIVYA TRIPATHI	OTHER	1	0.5	0.6	0.3	0.2	0.7	0.9	0.6
NEEDHI	XIAOMI	0.7	0.3	0	0.5	0.6	0.9	0.4	0.2
SHAGUFTA	SONY	1	0.5	0.6	0.3	0.2	0.7	0.9	0.6
RASHMI	APPLE	0.5	0.6	0.4	0.6	0.9	0.4	0.5	0.8
VEENA JOSHI	SAMSUNG	02	0.3	0.8	0.5	0.7	0.61	0	0.9
ARIHANT JAIN	OPPO	0.2	0.8	0.3	0.4	0.1	0.6	0.3	0.9
NIGHAT	REALME	0	0.8	0.7	0.9	1	0.5	0.7	0.5
IRFAN AH.	GOOGLE	0.7	0.2	0.6	0.8	0	0.1	0.4	0.5

Conflict of Interest

The authors declare no conflict of interest.

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Limitation of the Proposed Technique

It is necessary for max-min function operation to be defined on two matrices; the number of columns of one matrix should be equal to the number of rows of another matrix. Otherwise max-min operation i.e., max {min (a_{ij}, b_{ij})} will not be defined.

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